HYPERCOMPUTATION

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Contents

1 Terminology

Hypercomputer A computer that can compute anything that a Turing machine can compute, and more.

Hypercomputation The field that studies hypercomputers.

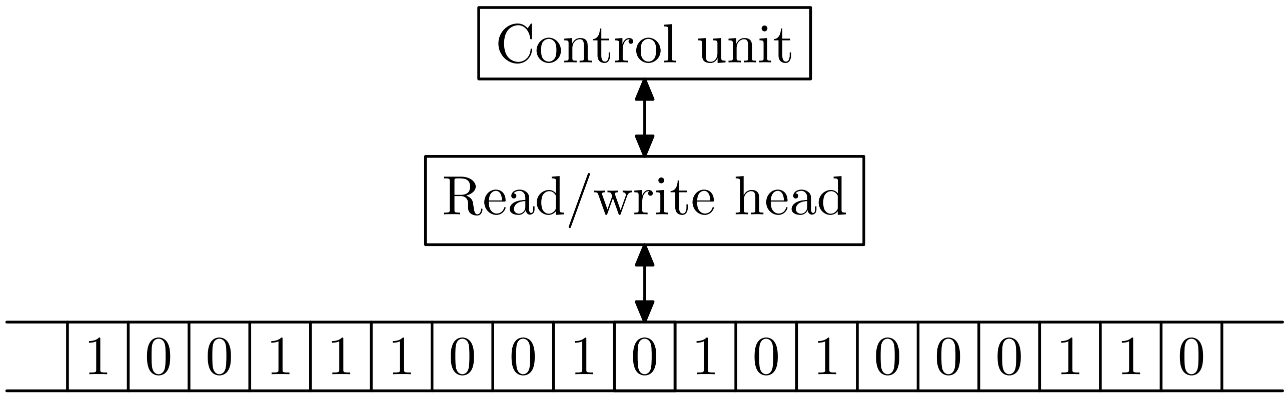
Hypercomputationalism The thesis that hypercomputers are of more than just logical and mathematical interest.

See `http://www.hypercomputation.net` for a list of currently active hypercomputationalists.

2 Recent Work on Hypercomputation

- Copeland, B.J. (2000) Narrow Versus Wide Mechanism. *Journal of Philosophy* 97, pp. 1–32.
- Copeland, B.J. & D. Proudfoot (1999) Alan Turing's Forgotten Ideas in Computer Science. *Scientific American* 280 (April 1999), pp. 99–103.
- Copeland, B.J. & R. Sylvan (1999) Beyond the Universal Turing Machine. *Australasian Journal of Philosophy* 77, pp. 46–66.
- Hava T. Siegelmann (1998) *Neural Networks and Analog Computation: Beyond the Turing Limit.*
- Copeland, B.J. (1997) The Broad Conception of Computation. *American Behavioral Scientist* 40, pp. 690–716.

3 The Turing Machine (1936)



A Turing machine consists of

- a control unit that has a finite number of possible states
- a read/write head that can read/write one symbol at a time
- a tape divided into squares; each square contains one symbol from a finite alphabet

Moreover, the behaviour of the control unit is describable by means of a finite list of quintuples of the form ⟨currently read symbol, current state, symbol to be written, next state, action⟩, where 'action' stands for 'go one square to the left' or 'go one square to the right'.

The machine stops if the control unit enters a designated halt state or encounters a symbol which it cannot handle in its current state.

A 'universal Turing machine' can simulate any Turing machine.

See Boolos and Jeffrey, *Computability and Logic*, for details.

4 Hypercomputers

4.1 O-Machines

Turing machines with an ‘oracle’ (Turing 1939).

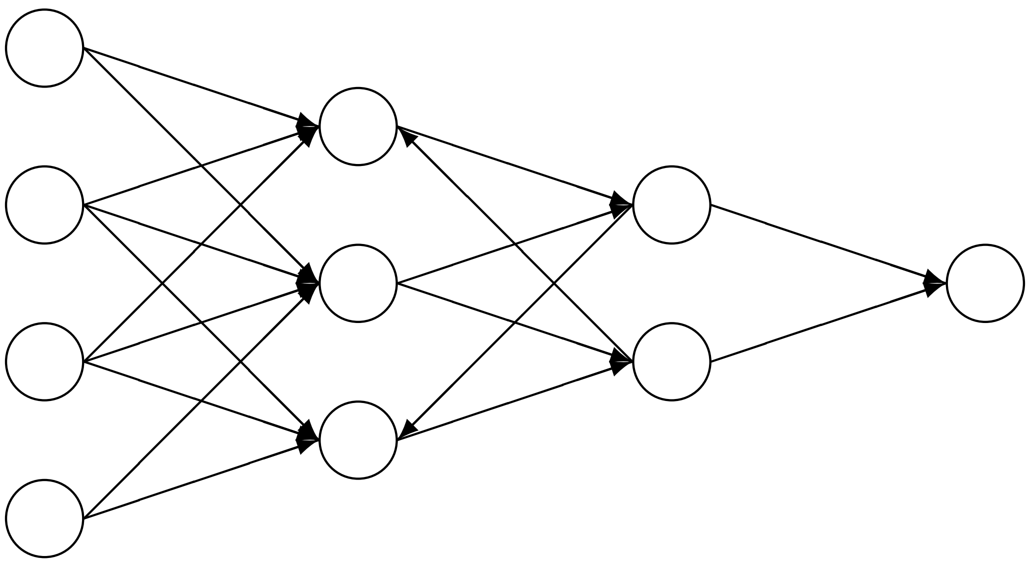
The oracle answers ‘yes/no’ questions, e.g., ‘Does Turing machine X halt on input Y?’

A universal Turing machine cannot answer this question.

An Interesting Class of O-Machines

Hava Siegelmann's analog recurrent neural networks with real-valued weights.

A simple neural net:



Siegelmann's results concerning analog recurrent neural networks:

nets with integer weights	finite automata
nets with rational weights	Turing machines
nets with real weights	O-machines

Basic idea: one of the weights in a real-valued net may be uncomputable.

A suitable decoder might extract interesting information from such a weight.

For example, let $w = 0.010111011000\ldots$ in binary notation.

Given an input string of length 3, the decoder might read off the 3rd digit of the binary expansion of w.

This might be the answer to the question whether the 3rd Turing machine halts given the 3rd input.

Is the Brain a Hypercomputer?

The neurophysiologically most adequate models of brain activity which exist today are the so-called “third generation” neural network models with **spiking neurons**.

These networks are equivalent with Siegelmann’s analog recurrent networks with real weights.

This raises the question: *Is the brain a hypercomputer?*

Well, is the brain a hypercomputer?

Probably not.

There is one factor that spoils the fun: noise.

Spiking neural nets subject to realistic types of noise are, in general, less powerful than finite automata (Wolfgang Maass & Pekka Orponen).

The same result applies to Siegelmann's super-Turing nets as well.

Other O-Machines

Some hypercomputationalists have speculated that there are entities in Nature which might be used in the same way as the uncomputable weights w of Siegelmann's nets.

Candidates that have been proposed:

- the physical constants (e.g., the gravitational constant G)
- the amount of background noise (from cosmic radiation and so on) at a given place as a function of time

My reaction: these phenomena may well be uncomputable, but it would be highly surprising if they could be used to build oracles.

Nevertheless, *The Noise Factory* (UK) has announced a hypercomputer (available within a few years) based on this idea.

4.2 Accelerated Turing Machines

The basic idea predates that of the Turing machine (R.M. Blake 1926, H. Weyl 1927).

First step: $\frac{1}{2}$ sec., second step: $\frac{1}{4}$ sec., n-th step: 2^{-n} sec. Since $\sum_{n=1}^{\infty} 2^{-n} = 1$, the machine may carry out infinitely many steps in one second.

An accelerated universal Turing machine can, in a sense, solve the problem 'Does Turing machine X halt on input Y?'

Let it simulate X with input Y and set it in motion.

Inspect its behaviour after 1 sec.

The machine will have halted (at some moment $t < 1$) if and only if the answer to the question is 'yes'.

Copeland is fond of accelerated Turing machines, but they are physically unrealistic.

The smallest measurement of length with any physical meaning is the Planck length (ca. 4×10^{-35}m, i.e., about 10^{-20} times the size of a proton).

The smallest measurement of time that has any physical meaning is the Planck time, the time it takes to cross the Planck length at the speed of light (ca. 1.4×10^{-43}s).

The 143rd step of the accelerated Turing machine takes 2^{-143} seconds, which is less than 10^{-43} seconds. Operations which are that fast make no sense from a physical point of view.

Even if we give the machine 4.5×10^9 years instead of just one second, it will already reach the 'Planck limit' at the 200th step.

4.3 Analog Automata

Several proposals.

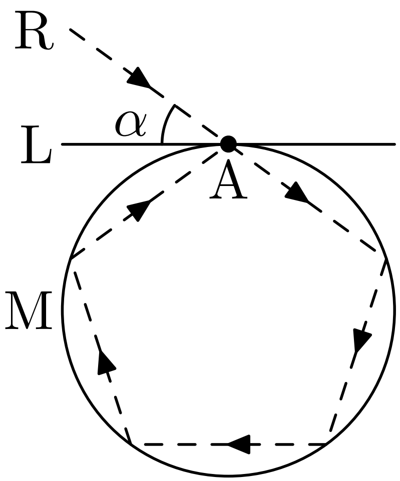
- Analog analogues of the Turing machine.

  L. Blum, F. Cucker, M. Shub and S. Smale (1997) *Complexity and Real Computation.*

- Extensions of the so-called General Purpose Analog Computer (Cris Moore and others).

  Problem: the new components are not realistic in "a world with noise, quantum effects, finite accuracy and limited resources."

An Illustration A simple analog apparatus capable of doing something that no Turing machine can do (after F. Waismann 1959).



M is a circular mirror with a reflecting surface on the inside; A is a small hole in M with a semi-transparent detector (light may come in but not go out); R is an incoming ray of light; α is the angle between R and the horizontal plane L.

By the laws of geometry and optics, the detector at A gets hit from the inside if and only if there is some rational number q such that $\alpha = q \times \pi$. (In the diagram, $\alpha = \frac{1}{5}\pi$.)

Thus, the apparatus determines whether α is a rational multiple of π.

No Turing machine can perform this calculation!

An Illustration. . . (Continued) This apparatus illustrates what is wrong with analog computers in general.

It does not work unless A is *infinitely small* and M is *perfectly* circular.

Moreover, we can never be sure that α is *exactly* what we want.

4.4 Other Ideas

Interesting Turing machines that accept (possibly uncomputable) input (whose tape gets changed) as they work.

Fanciful Machines that communicate with other universes with different physical laws (e.g., vastly different universes in which *Newtonian physics* instead of Relativity Theory and Quantum Mechanics holds).

5 Against Hypercomputationalism

5.1 Objection 1. The Bekenstein Bound

Theorem A spherical region with radius R and energy E can contain **only a limited amount of information** I (in the sense of number of distinguishable quantum states):

$$I \leq 2\pi ER/\hbar c \ln 2$$

where $\hbar$ is Planck's constant and c is the speed of light.

The time for a state transition cannot be less than the time it takes for light to cross a sphere of radius R, which is $2R/c$.

So the Bekenstein bound implies that there is a **maximum information processing rate** N:

$$N \leq \pi E/\hbar \ln 2$$

The Bekenstein Bound entails that Turing machines—with their infinite number of configurations—are physically impossible (at least if they are to have a finite size and bounded energy).

The same applies *a fortiori* to Hypercomputers.

Only certain types of finite automata are physically possible.

Possible Counterargument Even a two-state device may display uncomputable behaviour, e.g., a totally random succession of occurrences of state_0 and state_1.

Does QM have anything to say about this?

5.2 Objection 2. Empirical Meaningfulness

Even if a hypercomputer were put into our hands, we could not determine whether it is a hypercomputer.

For we can only make a *finite* number of observations of *limited* precision.

And any finite collection of finite data can be accounted for by assuming that they were produced by a finite automaton.

So the claim that a given device is a hypercomputer rather than a Turing machine—or a Turing machine rather than a finite automaton—is in a sense *empirically meaningless*.

Possible Counterargument This objection applies to all scientific theories.

They always go beyond the data.

The hypothesis that a certain apparatus is a hypercomputer does not differ from any other scientific hypothesis in this respect.

6 Conclusion

It remains to be seen if the field of hypercomputation is of more than merely logical and mathematical interest.

Anyway, the field certainly broadens our perspective.

As Copeland and Sylvan have written:

We would be profoundly surprised if the physics of the real world can be properly and fully set out without departing from the set of Turing-machine-computable functions.

These functions have been the focus of intense interest during the brief six decades since Turing delineated them, but the explanation of this is surely their extreme tractability, together, of course, with the fact that they have made a considerable number of people very rich, rather than because some inherent suitability for exhaustively describing the structure and properties of matter is discernible in them.

Moreover, as we have already related, these functions were the fruit of Turing's analysis of the activity of an idealised human mathematician working mechanically with pencil and paper.

It is simple anthropomorphism to expect the same set of functions to be prominent in the behaviour of the world minus human mathematicians.

In short it would—or should—be one of the greatest astonishments of science if the activity of Mother Nature were never to stray beyond the bounds of Turing-machine-computability.

7 For Further Reading

- `http://www.hypercomputation.net`
- Jack Copeland and Gert-Jan Lokhorst, Hypercomputation: A Dialogue, *Minds and Machines*, special issue on Hypercomputation, to appear in 2001 or 2002.
- These transparencies are available at
  `http://www.eur.nl/fw/staff/lokhorst/hypercomputation.helsinki.html`