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Ernst Mally's *Deontik* (1926)

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Abstract In 1926, Mally proposed the first formal deontic system. As Mally and others soon realized, this system had some rather strange consequences. We show that the strangeness of Mally's system is not so much due to Mally's informal deontic principles as to the fact that he formalized those principles in terms of the propositional calculus. If they are formalized in terms of relevant logic rather than classical logic, one obtains a system which is related to Anderson's relevant deontic logic and not nearly as strange as Mally's own system.

1 Introduction Mally, a student of Meinong, was the first philosopher to build a formal theory of normative concepts [7]. It would be an understatement to say that Mally's *Deontik* is not held in high esteem today. Føllesdal and Hilpinen [6] call it "strange," "counter-intuitive," and "not acceptable" and mainly wonder "Where did Mally go wrong?" Meyer and Wieringa [10] mention it only "by way of curiosity" before proceeding to what they describe as "the first 'real' system of deontic logic." In this paper, we will show that the strangeness of Mally's system is not so much due to his basic deontic assumptions as to the fact that he formalized those assumptions in terms of the propositional calculus. If they are formalized in terms of relevant logic rather than classical logic, one obtains a system which is related to Anderson's relevant deontic logic [1] and not nearly as strange as Mally's own system.

2 Mally's **Deontik** Mally adopted the following informal deontic principles ([7], §2):

1. If A requires B and if B then C, then A requires C.
2. If A requires B and if A requires C, then A requires B and C.
3. A requires B if and only if it is obligatory that if A then B.
4. The unconditionally obligatory is obligatory.
5. The unconditionally obligatory does not require its own negation.

In order to formalize these principles, Mally used the propositional calculus **PC** supplemented with a propositional constant U, which he read as 'the unconditionally

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obligatory' (*das unbedingt Geforderte*) and with a monadic propositional operator ! which he read as 'it is obligatory that' (he read $!A$ as '*es sei A*' or as '*A soll sein*'). He translated 'A requires B' as $A \supset !B$. As a result he obtained the following axioms:

MD1	$((A \supset !B) \wedge (B \supset C)) \supset (A \supset !C)$
MD2	$((A \supset !B) \wedge (A \supset !C)) \supset (A \supset !(B \wedge C))$
MD3	$(A \supset !B) \equiv !(A \supset B)$
MD4	$!U$
MD5	$\neg(U \supset !\neg U)$

Mally formalized (4) as $\exists U!U$. We agree with Føllesdal and Hilpinen ([6], pp. 2–3) that this formula is not well-formed and should be replaced by $!U$. Mally viewed the latter formula as a theorem ([7], §3, formula 15).

Mally's principles sound more or less natural but many of the theorems of his system are decidedly strange. For example, one may prove the following theorem:

(†) $\quad !A \equiv A$

Proof: We have $((!A \supset !A) \wedge (A \supset B)) \supset (!A \supset !B)$ in virtue of MD1, whence $(A \supset B) \supset (!A \supset !B)$ by **PC**. (For the rest of the proof, see [6], p. 4, formulas 9–21.) (†) implies that Mally's system is trivial. It has no genuine deontic content. The deontic operator ! has a purely decorative function and can be defined away by $!A = A$. □

Although he did not actually discuss (†), Mally was well aware of the fact that his theory had many strange consequences. Indeed, he classified thirteen of the thirty-five theorems which he derived as *befremdlich*. Mally defended these strange theorems, but all later deontic logicians (starting with Menger [9]) have regarded them as fatal to his theory.

In their well known account of Mally's system, Føllesdal and Hilpinen added one additional rule of inference: $A \equiv B \;/\; !A \equiv !B$ ([6], p. 3). This rule is, however, derivable from $(A \equiv B) \supset (!A \equiv !B)$ which is a theorem in virtue of MD1, so there is no need to introduce it as a primitive rule. These authors also added the following axiom: $!A \equiv \forall P(P \supset !A)$ ([6], p. 3). But if propositional quantifiers are introduced in the usual way, then $A \equiv \forall P(P \supset A)$ will be a theorem, so $!A \equiv \forall P(P \supset !A)$ will also be a theorem. There is therefore no reason to introduce this formula as an axiom.

3 Relevant logic Where did Mally go wrong? Like Føllesdal and Hilpinen, we suspect that the unacceptability of his system is not so much due to his informal deontic principles as to the fact that he formalized those principles in terms of the propositional calculus. Føllesdal and Hilpinen suggested that Mally should perhaps have used strict implication instead of material implication. They did not explore this idea and we will not do so either. Rather, we will investigate the consequences of replacing Mally's material implication by relevant implication as formalized in the Anderson-Belnap logic of relevant implication **R**. **R** is defined as follows ([3], chap. 5).

Syntax $\quad$ A is a formula if and only if: (i) A is an atomic sentence or (ii) B and C are formulas and $A = \neg B$ or $A = (B \to C)$ or $A = (B \,\&\, C)$ or $A = (B \vee C)$. Outer parentheses will usually be omitted.

Definitions $A \longleftrightarrow B = (A \to B) \,\&\, (B \to A)$, $A \circ B = \neg(A \to \neg B)$, $A + B = \neg A \to B$.

Axioms and rules

Self-implication	$A \to A$
Prefixing	$(A \to B) \to ((C \to A) \to (C \to B))$
Contraction	$(A \to (A \to B)) \to (A \to B)$
Permutation	$(A \to (B \to C)) \to (B \to (A \to C))$
&Elim	$(A \,\&\, B) \to A$, $(A \,\&\, B) \to B$
&Int	$((A \to B) \,\&\, (A \to C)) \to (A \to (B \,\&\, C))$
∨Int	$A \to (A \vee B)$, $B \to (A \vee B)$
∨Elim	$((A \to C) \,\&\, (B \to C)) \to ((A \vee B) \to C)$
Distribution	$(A \,\&\, (B \vee C)) \to ((A \,\&\, B) \vee C)$
Double negation	$\neg\neg A \to A$
Contraposition	$(A \to \neg B) \to (B \to \neg A)$
Modus Ponens	$A, A \to B \;/\; B$
Adjunction	$A, B \;/\; A \,\&\, B$

System $\mathbf{R}^{Ftu}$ is defined as follows.

Syntax The same as that of **R**, except that there are three propositional constants, F, t, and u (F and t are discussed in [3], §27.1.2). F is 'the conjunction of all propositions', t is 'the conjunction of all truths', u is 'the unconditionally obligatory'.

Axioms The same as those of **R**, plus

RF	$F \to A$
Rt	$A \longleftrightarrow (t \to A)$

$\mathbf{R}^{Ftu}$ is a conservative extension of **R** in the sense that all theorems of $\mathbf{R}^{Ftu}$ which contain no occurrences of F, t, and u are theorems of **R**. The term 'system **R**' will from now on refer to system $\mathbf{R}^{Ftu}$ rather than **R**.

In the following, we will occasionally refer to the following theorems and derived rule of inference of **R** (the proofs are left to the reader):

&Importation	$(A \to (B \to C)) \to ((A \,\&\, B) \to C)$
Contraposition′	$(A \to B) \longleftrightarrow (\neg B \to \neg A)$
MP′	$A \to B, B \to C \;/\; A \to C$

***4 Relevant* Deontik** Our first system of Relevant *Deontik*, **RD**, is defined as follows.

Language A is a formula if and only if: (i) A is a formula of the language of **R**, or (ii) B is a formula and $A = OB$. OA is read as 'it is obligatory that A'.

Alethic axioms and rules System **R**.

Deontic axioms (compare axioms MD1–MD5 above)

RD1 $((A \to OB) \,\&\, (B \to C)) \to (A \to OC)$
RD2 $((A \to OB) \,\&\, (A \to OC)) \to (A \to O(B \,\&\, C))$
RD3 $(A \to OB) \longleftrightarrow O(A \to B)$
RD4 Ou
RD5 $\neg(u \to O\neg u)$

Our second system of Relevant *Deontik*, **RD***, is defined in the same way as **RD** except that there is one additional axiom:

RD6 $OA \to (u \to A)$

The formula $!A \supset (U \supset A)$—the counterpart of RD6 in Mally's language—does not occur in Mally's book (but see Theorem 18′ below). $!A \supset (U \supset A)$ is equivalent with $!A \supset (\neg A \supset \Omega)$, where Ω is the unconditionally forbidden. According to Anderson ([1], p. 348), the latter formula was first discussed by Bohnert in 1945 [4].

Theorem 4.1 *RD6 is not a theorem of **RD**.*

Proof: Define a function $\mathcal{T}$ from the language of **RD** into the language of **R** as follows: $\mathcal{T}(a) = a$, where a is an atomic sentence, $\mathcal{T}(\neg A) = \neg\mathcal{T}(A)$, $\mathcal{T}(A \star B) = \mathcal{T}(A) \star \mathcal{T}(B)$, where $\star$ is $\to$, &, or $\vee$, $\mathcal{T}(F) = F$, $\mathcal{T}(t) = t$, $\mathcal{T}(u) = p \to p$, where p is some atomic sentence, $\mathcal{T}(OA) = \mathcal{T}(A)$. $\mathcal{T}$ transforms all axioms of **RD** into theorems of **R**.

Proof of $\mathcal{T}$(RD1)*:*

1. $(A \to B) \to ((B \to C) \to (A \to C))$ Pref, Perm, MP
2. $((A \to B) \,\&\, (B \to C)) \to (A \to C)$ 1, &Imp, MP
3. $\mathcal{T}(((A \to OB) \,\&\, (B \to C)) \to (A \to OC))$ 2, Def $\mathcal{T}$

Proof of $\mathcal{T}$(RD2)*:* From &Int and Def $\mathcal{T}$.

Proof of $\mathcal{T}$(RD3)*:* From Self-impl, Adj, and Def $\mathcal{T}$.

Proof of $\mathcal{T}$(RD4)*:* From Self-impl and Def $\mathcal{T}$.

Proof of $\mathcal{T}$(RD5)*:*

1. $((p \to p) \to \neg(p \to p)) \to \neg(p \to p)$ *Reductio* [3], §14.1.3
2. $(p \to p) \to \neg((p \to p) \to \neg(p \to p))$ 1, Contrapos, MP
3. $\neg((p \to p) \to \neg(p \to p))$ 2, Self-impl, MP
4. $\mathcal{T}(\neg(u \to O\neg u))$ 3, Def $\mathcal{T}$

$\mathcal{T}$ also transforms all rules of **RD** into rules of **R**. So $\mathcal{T}$ transforms all theorems of **RD** into theorems of **R**. $\mathcal{T}(\text{RD6}) = A \to ((p \to p) \to A)$ which is certainly not a theorem of **R**, so RD6 is not a theorem of **RD** which was to be proven. □

5 Anderson's relevant deontic logic Anderson's relevant deontic logic, **AL**, is defined as follows ([1], [2], [8]).

Language	The same as that of **R**, with the following additional definition: $OA = u \rightarrow A$. Anderson read u as "the good thing" ([2], p. 277). (He actually took a propositional constant V ("the bad thing") as primitive and defined u as $\neg V$. We could, of course, define V as $\neg u$.)
Axioms and rules	$\mathbf{R} + u \circ u$. $u \circ u$ may also be written as $\neg O \neg u$ in virtue of Anderson's definition of O. This axiom is known as the 'Axiom of Avoidance' (see [2], p. 280 and [8], pp. 146–48).

We will be not so much interested in **AL** as in a weaker system **AL**$'$, which is defined in the same way as **AL** except that the Axiom of Avoidance is replaced by $u \circ u \circ u$. This axiom is provably equivalent with $\neg(u \rightarrow O\neg u)$ in virtue of Def O and **R**. **AL**$'$ is weaker than **AL** because $u \circ u \rightarrow u \circ u \circ u$ is a theorem of **R** whereas $u \circ u$ is not a theorem of $\mathbf{R} + u \circ u \circ u$.

6 *RD** and *AL*$'$

Theorem 6.1 ***RD**** *and* ***AL***$'$ *have exactly the same theorems.*

Proof: We first show that all theorems of **RD*** are theorems of **AL**$'$. Because all rules of inference of **RD*** are rules of **AL**$'$, it is sufficient to prove that all axioms of **RD*** are theorems of **AL**$'$.

1.	$(B \rightarrow C) \rightarrow ((u \rightarrow B) \rightarrow (u \rightarrow C))$	Pref
2.	$(B \rightarrow C) \rightarrow (OB \rightarrow OC)$	2, Def O
3.	$(OB \rightarrow OC) \rightarrow ((A \rightarrow OB) \rightarrow (A \rightarrow OC))$	Pref
4.	$(B \rightarrow C) \rightarrow ((A \rightarrow OB) \rightarrow (A \rightarrow OC))$	2, 3, MP$'$
5.	$(A \rightarrow OB) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow OC))$	4, Perm, MP
RD1	$((A \rightarrow OB) \,\&\, (B \rightarrow C)) \rightarrow (A \rightarrow OC)$	5, &Imp, MP

1.	$((u \rightarrow B) \,\&\, (u \rightarrow C)) \rightarrow (u \rightarrow (B \,\&\, C))$	&Int
2.	$(OB \,\&\, OC) \rightarrow O(B \,\&\, C)$	1, Def O
3.	$((A \rightarrow OB) \,\&\, (A \rightarrow OC)) \rightarrow (A \rightarrow (OB \,\&\, OC))$	&Int
4.	$(A \rightarrow (OB \,\&\, OC)) \rightarrow (A \rightarrow O(B \,\&\, C))$	2, Pref, MP
RD2	$((A \rightarrow OB) \,\&\, (A \rightarrow OC)) \rightarrow (A \rightarrow O(B \,\&\, C))$	3, 4, MP$'$

1.	$(A \rightarrow (u \rightarrow B)) \rightarrow (u \rightarrow (A \rightarrow B))$	Perm
2.	$(u \rightarrow (A \rightarrow B)) \rightarrow (A \rightarrow (u \rightarrow B))$	Perm
3.	$(A \rightarrow (u \rightarrow B)) \longleftrightarrow (u \rightarrow (A \rightarrow B))$	1, 2, Adj
RD3	$(A \rightarrow OB) \longleftrightarrow O(A \rightarrow B)$	3, Def O

1.	$u \rightarrow u$	Self-impl
RD4	Ou	1, Def O

1.	$u \circ u \circ u$	Axiom

2.	$\neg(u \to (u \to \neg u))$	1, Contrapos′, MP
RD5	$\neg(u \to O\neg u)$	2, Def O

1.	$(u \to A) \to (u \to A)$	Self-impl
RD6	$OA \to (u \to A)$	1, Def O

With this we have proven that all theorems of **RD*** are theorems of **AL**′.

We now show that all theorems of **AL**′ are theorems of **RD***. We first show that $(u \to A) \to OA$ is a theorem of **RD**.

1.	$Ou \to Ou$	Self-impl
2.	$(u \to A) \to (u \to A)$	Self-impl
3.	$u \to ((u \to A) \to A)$	2, Perm, MP
4.	$(Ou \to Ou)\ \&\ (u \to ((u \to A) \to A))$	1, 3, Adj
5.	$Ou \to O((u \to A) \to A)$	4, RD1, MP
6.	$O((u \to A) \to A)$	5, RD4, MP
7.	$(u \to A) \to OA$	6, RD3, &Elim, MP

In conjunction with RD6 this result implies that $OA \longleftrightarrow (u \to A)$ is a theorem of **RD***. Furthermore, **RD*** has the following theorem:

Replacement $\quad ((A \longleftrightarrow B)\ \&\ (C \to C)) \to (C \longleftrightarrow C[A\ /\ B])$,

where $C[A\ /\ B]$ is the result of replacing zero or more occurrences of A in C by B (proof: by induction on the length of C; see [5], §1.6 for some details). Using these facts we may prove that if A is a theorem of **AL**′, A is a theorem of **RD***. The proof is by induction on the length of a proof of A in **AL**′. All cases are obvious, except one. Suppose that Def O is used to derive $A[OA\ /\ u \to A]$ or $A[u \to A\ /\ OA]$ from A. One may then use $OA \longleftrightarrow (u \to A)$, Replacement, and MP to carry out the same derivation in **RD***. Thus any proof of A in **AL**′ is replicable in **RD***. This implies that each theorem of **AL**′ is a theorem of **RD***. This ends the proof of Theorem 6.1. □

In sum, the sets of theorems of the four relevant deontic systems we have discussed are related as follows:

$$\mathrm{Th}(\mathbf{RD}) \subset \mathrm{Th}(\mathbf{RD^*}) = \mathrm{Th}(\mathbf{AL}') \subset \mathrm{Th}(\mathbf{AL}).$$

Note that neither $A \to OA$ nor $OA \to A$ is a theorem of **AL** because neither $A \to (u \to A)$ nor $(u \to A) \to A$ is a theorem of $\mathbf{R} + u \circ u$. It follows from this that **RD**, **RD***, **AL**′, and **AL** are not trivial.

The above may be summarized as follows: if you add $OA \to (u \to A)$ to Relevant *Deontik* and strengthen $\neg(u \to O\neg u)$ to $\neg O\neg u$, then you get Anderson's system.

7 Mally's* Deontik *revisited In [7], Sections 4–7, Mally discussed thirty-five theorems of his *Deontik*. He classified thirteen of these theorems as "strange" (*befremdlich*). He apparently regarded the other twenty-two theorems as "plausible."

Let $\mathcal{T}_X(A)$ denote the translation of A into the language of system X. Let us say that A agrees with Mally's pretheoretical intuitions from the perspective of system

X if and only if either (i) A is "plausible" according to Mally and $\mathcal{T}_X(A)$ is derivable in X, or (ii) A is "strange" according to Mally and $\mathcal{T}_X(A)$ is not derivable in X. Let $\mathcal{N}(X)$ denote the number of formulas on Mally's list of theorems which agree with Mally's intuitions from the perspective of system X. Let us say that system X is in better accordance with Mally's intuitions than system Y if and only if $\mathcal{N}(X) > \mathcal{N}(Y)$. And let **MD** denote Mally's original *Deontik*. In this section we will show that $\mathcal{N}(\mathbf{MD}) = 22$, $\mathcal{N}(\mathbf{RD}) = 25$, and $\mathcal{N}(\mathbf{RD^*}) = 27$. So we have the following theorem.

Theorem 7.1 ***RD*** *is in better accordance with Mally's pretheoretical intuitions than his own system* ***MD*** *was, and* ***RD**** *is even better.*

Proof: We use the following table to translate Mally's formulas into the language of **RD**:

MD	**RD**	**MD**	**RD**	**MD**	**RD**
$\supset$	$\rightarrow$	$\forall M(M \supset A)$	$t \rightarrow A$	U	u
$!A$	OA	$\forall M(A \supset OM)$	$A \rightarrow OF$	Π	$\neg u$
AfB	$A \rightarrow OB$	$\wedge$	$\&$	V	t
$A \infty B$	$O(A \longleftrightarrow B)$	$\vee$	$\vee$	Λ	$\neg t$

In Theorems 12 and 13 of List I below, $\wedge$ will be translated by $\circ$ and $\vee$ by $+$.

We divide Mally's theorems into five categories for the sake of clarity.

7.1 List I The following list includes all "plausible" theorems of **MD** whose translations are theorems of **RD**.

3 $((MfA) \vee (MfB)) \supset (Mf(A \vee B))$
4 $((MfA) \,\&\, (NfB)) \supset ((M \,\&\, N)\, f(A \,\&\, B))$
5 $!A \equiv \forall M(MfA)$
6 $(!A \wedge (A \rightarrow B)) \supset\, !B$
8 $((AfB) \,\&\, (BfC)) \supset (AfC)$
9 $(!A \wedge (AfB)) \supset\, !B$
10 $(!A \wedge !B) \equiv\, !(A \wedge B)$
11 $(A \infty B) \equiv\, !(A \equiv B)$
12 $(AfB) \equiv (A \supset !B) \equiv\, !(A \supset B) \equiv\, !\neg(A \wedge \neg B) \equiv\, !(\neg A \vee B)$
13 $(A \supset !B) \equiv \neg(A \wedge \neg !B) \equiv (\neg A \vee !B)$
14 $(AfB) \equiv (\neg B f \neg A)$
15 $\forall M(MfU)$
16 $(U \supset A) \supset\, !A$

The translations of these formulas are as follows. The numbers of translated formulas are underlined.

$\underline{3}$ $((A \to OB) \vee (A \to OC)) \to (A \to O(B \vee C))$
$\underline{4}$ $((A \to OB) \,\&\, (C \to OD)) \to ((A \,\&\, C) \to O(B \,\&\, D))$
$\underline{5}$ $OA \longleftrightarrow (t \to OA)$
$\underline{6}$ $(OA \,\&\, (A \to B)) \to OB$
$\underline{8}$ $((A \to OB) \,\&\, (B \to OC)) \to (A \to OC)$
$\underline{9}$ $(OA \,\&\, (A \to OB)) \to OB$
$\underline{10}$ $(OA \,\&\, OB) \longleftrightarrow O(A \,\&\, B)$
$\underline{11}$ $O(A \longleftrightarrow B) \longleftrightarrow O(A \longleftrightarrow B)$
$\underline{12}$ $(A \to OB) \longleftrightarrow (A \to OB) \longleftrightarrow O(A \to B) \longleftrightarrow O\neg(A \circ \neg B) \longleftrightarrow O(\neg A + B)$
$\underline{13}$ $(A \to OB) \longleftrightarrow \neg(A \circ \neg OB) \longleftrightarrow (\neg A + OB)$
$\underline{14}$ $(A \to OB) \longleftrightarrow (\neg B \to O\neg A)$
$\underline{15}$ $t \to Ou$
$\underline{16}$ $(u \to A) \to OA$

The **RD**-proofs of the translated formulas are generally easy. We only give a hint for $\underline{8}$: the antecedent is provably equivalent with $O((A \to B) \,\&\, (B \to C))$. $\underline{16}$ was already proven in the proof of Theorem 6.1.

7.2 List II The following list includes all "plausible" theorems of **MD** whose translations do not seem to be derivable in **RD** but which are derivable in **RD***.

17	$(UfA) \supset\, !A$	$\underline{17}$	$(u \to OA) \to OA$
18	$!!A \supset\, !A$	$\underline{18}$	$OOA \to OA$

Mally derived 17 from 8 and MD4 but the corresponding derivation does not go through in **RD**. Mally derived 18 from 17 with the help of the following principle which he mentioned only informally:

18′	$!!A \supset (UfA)$	$\underline{18'}$	$OOA \to (u \to OA)$

$\underline{18'}$ is definitely not a theorem of **RD** (proof: as in the proof of Theorem 4.1). $\underline{17}$ and $\underline{18}$ are theorems of **RD*** in virtue of Contraction, RD6, and $\underline{16}$. $\underline{18'}$ is a theorem of **RD*** because it is an instance of RD6.

7.3 List III The following list includes all "plausible" theorems whose translations are not derivable in **RD***.

19	$!A \equiv\, !!A$	$\underline{19}$	$OA \longleftrightarrow OOA$
20	$(UfA) \equiv (A \infty U)$	$\underline{20}$	$(u \to OA) \longleftrightarrow O(A \longleftrightarrow u)$
21	$!A \equiv (A \infty U)$	$\underline{21}$	$OA \longleftrightarrow O(A \longleftrightarrow u)$
23	$V \infty U$	$\underline{23}$	$O(t \longleftrightarrow u)$
24	AfA	$\underline{24}$	$A \to OA$
25	$(A \supset B) \supset (AfB)$	$\underline{25}$	$(A \to B) \to (A \to OB)$
26	$(A \equiv B) \supset (A \infty B)$	$\underline{26}$	$(A \longleftrightarrow B) \to O(A \longleftrightarrow B)$

The translated formulas are not derivable in **RD*** because they are not derivable in **AL**′ as is easily checked. For example, in **AL**′, $\underline{19}$ is an abbreviation of $(u \to A) \longleftrightarrow (u \to (u \to A))$, which is not a theorem of $\mathbf{R} + u \circ u \circ u$.

Most, if not all, of the just-presented seven theorems are less "plausible" than Mally thought. For example, the "strange" formula 22 below is an immediate consequence of V (an axiom of Mally's system), theorem 24, and modus ponens. If 22 is strange, then 24 should be regarded as strange too. But then 19, 23, 25, and 26, which Mally derived from 22 and 24, should not be accepted without hesitation either.

7.4 List IV The following list includes all ‘strange’ theorems whose translations are not derivable in **RD***.

1	$(AfB) \supset (AfV)$	$\underline{1}$	$(A \to OB) \to (A \to Ot)$
2	$(Af\Lambda) \equiv \forall M(AfM)$	$\underline{2}$	$(A \to O\neg t) \longleftrightarrow (A \to OF)$
7	$!A \supset\, !V$	$\underline{7}$	$OA \to Ot$
22	$!V$	$\underline{22}$	Ot
27	$\forall M(\cap fM)$	$\underline{27}$	$\neg u \to OF$
28	$\cap f \cap$	$\underline{28}$	$\neg u \to O\neg u$
29	$\cap fU$	$\underline{29}$	$\neg u \to Ou$
31	$\cap \infty \Lambda$	$\underline{31}$	$O(\neg u \longleftrightarrow \neg t)$
32	$\neg(Uf\Lambda)$	$\underline{32}$	$\neg(u \to O\neg t)$
33	$\neg(U \supset \Lambda)$	$\underline{33}$	$\neg(u \to \neg t)$
34	$U \equiv V$	$\underline{34}$	$u \longleftrightarrow t$
35	$\cap \equiv \Lambda$	$\underline{35}$	$\neg u \longleftrightarrow \neg t$

7.5 List V There is only one formula left. Mally regarded it as strange, but its translation is a theorem of **RD**.

30	$Uf\Lambda$	$\underline{30}$	$\neg u \to O\neg t$

$\underline{30}$ is a theorem of **RD** because we have $\neg u \to (u \to \neg t)$ in virtue of Rt and Contraposition, whence $\neg u \to O\neg t$ by $\underline{16}$.

Taking stock, we see that $\mathcal{N}(\mathbf{MD}) = 22$ (Lists I–III), $\mathcal{N}(\mathbf{RD}) = 25$ (Lists I and IV), and $\mathcal{N}(\mathbf{RD^*}) = 27$ (Lists I, II, and IV). So **RD** is in better accordance with Mally’s intuitions than his own system **MD** was, and **RD*** is even better. This ends the proof of Theorem 7.1. □

8 Four final observations First, **RD*** is in better accordance with Mally’s intuitions than **AL** (Anderson’s own relevant deontic logic) because the “strange” formula $\underline{32}$ is a theorem of **AL** in virtue of the Axiom of Avoidance (so $\mathcal{N}(\mathbf{AL}) = 26$).

Second, we are now in a position to state where Mally went wrong. In a *general* sense: by formalizing his deontic principles in terms of classical propositional logic. In a *specific* sense: at each theorem of **MD** whose translation is not derivable in **RD** (Lists II, III, and IV).

Third, Mally regarded 34 (and 35, which is just the contraposed version of 34) as the strangest of his strange theorems. From the perspective of **RD***, 1, 2, 7, 20, 21, 27, and 29 are even stranger because the translations of these formulas are not derivable in **RD*** + $\underline{34}$.

Fourth, (†), the strange consequence of Mally’s theory with which we started, is in a certain sense stranger than 34 because $OA \to A$ does not seem to be a theorem of **RD** + $\underline{34}$. (The converse formula, $A \to OA$, is a theorem in virtue of $\underline{16}$, $\underline{34}$, and Rt.) For although RD1 entails $((u \to OA) \,\&\, (A \to \neg u)) \to (u \to O\neg u)$, whence $\neg((u \to OA) \,\&\, (A \to \neg u))$ by RD5 and Contraposition, whence $\neg(OA \,\&\, \neg A)$ by $\underline{34}$ and Rt, the latter formula does not entail $OA \to A$. But if RD1 is strengthened to

$(A \to OB) \to ((B \to C) \to (A \to OC))$, then $OA \to A$ does become a theorem in the presence of 34. (The proof is similar to the proof of $\neg(OA \,\&\, \neg A)$.)

9 Conclusion We do not wish to defend any of the systems we have discussed. Mally's system is unacceptable because of its triviality. The relevant deontic systems are problematic for the reasons given in [8]. We only wanted to make it clear where Mally went wrong. It was his reliance on classical logic which led him into trouble. If Mally's ideas are expressed in terms of relevant logic rather than classical logic, we obtain a system which is similar to Anderson's relevant deontic logic and not nearly as strange as Mally's original system.

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