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Anderson's Relevant Deontic and Eubouliatic Systems

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Abstract We present axiomatizations of the deontic fragment of Anderson's relevant deontic logic (the logic of obligation and related concepts) and the eubouliatic fragment of Anderson's eubouliatic logic (the logic of prudence, safety, risk, and related concepts).

1 Introduction

In 1967, Anderson [2] defined his system of relevant deontic logic as follows: take relevant system **R**, add a propositional constant V ("the violation" or "the bad thing"), and define O ("it is obligatory that") by $OA = \neg A \to V$, where $\to$ is relevant implication. This proposal naturally leads to the question: to which purely deontic system, stated in terms of O rather than V, does this definition give rise? This problem is known as the problem of characterizing the *deontic fragment* of this system. This problem was solved by Goble [5], but his solution was long and complicated because it was based on the Routley-Meyer semantics of **R**.

In 1968, Anderson [3] defined his system of relevant eubouliatic logic as follows: take system **R**, add a constant G ("the good thing"), and define R_w ("it is without risk that," "it is safe that") by $R_w A = A \to G$. This proposal likewise raises the question: to which purely eubouliatic system, stated in terms of R_w rather than G, does this definition give rise? This is the problem of characterizing the *eubouliatic fragment* of this system. This problem has not yet been addressed in the literature, as far as we know. In this paper, we show that both problems may be solved by following a simple syntactic approach.

2 Anderson's Relevant Deontic Logic

Definition 2.1 Relevant system **R** is defined as follows.

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Axioms and rules:

R1	$A \to A$	self-implication
R2	$(A \to B) \to ((C \to A) \to (C \to B))$	prefixing
R3	$(A \to (B \to C)) \to (B \to (A \to C))$	permutation
R4	$(A \to (A \to B)) \to (A \to B)$	contraction
R5	$(A \& B) \to A$, $(A \& B) \to B$	&elimination
R6	$((A \to B) \& (A \to C)) \to (A \to (B \& C))$	&introduction
R7	$A \to (A \vee B)$, $B \to (A \vee B)$	$\vee$introduction
R8	$((A \to C) \& (B \to C)) \to ((A \vee B) \to C)$	$\vee$elimination
R9	$(A \& (B \vee C)) \to ((A \& B) \vee C)$	distribution
R10	$\neg\neg A \to A$	double negation
R11	$(A \to \neg B) \to (B \to \neg A)$	contraposition
$\to$E	$A, A \to B / B$	detachment
&I	$A, B / A \& B$	adjunction

Definition: $A \leftrightarrow B = (A \to B) \& (B \to A)$.

We mention the following theorems of **R** for later reference.

T1 $(A \to B) \to ((B \to C) \to (A \to C))$,
T2 $(A \to B) \to (\neg B \to \neg A)$.

T1 follows from prefixing and permutation. T2 can be proven as follows. $\vdash \neg\neg\neg B \to \neg B$ by double negation, hence $\vdash B \to \neg\neg\neg\neg B$ by contraposition, hence $\vdash B \to \neg\neg B$ by double negation and T1, hence $\vdash (A \to B) \to (A \to \neg\neg B)$ by prefixing and T1; $\vdash (A \to \neg\neg B) \to (\neg B \to \neg A)$ by contraposition; hence $\vdash (A \to B) \to (\neg B \to \neg A)$ by T1.

Definition 2.2 Deontic system $\mathbf{R}_O$ is **R** plus an operator O ("it is obligatory that") and the following axioms:

D1 $(A \to B) \to (OA \to OB)$,
D2 $O(OA \to A)$.

Definition 2.3 Deontic system $\mathbf{R}'_O$ is $\mathbf{R}_O$ plus a constant V ("the violation" or "the bad thing") and the following axioms:

D3 $OA \to (\neg A \to V)$,
D4 $(\neg A \to V) \to OA$.

We refer to those formulas of $\mathbf{R}'_O$ in which V occurs, if at all, only in contexts of the form $\neg A \to V$ (so that V is always eliminable in terms of O) as V-formulas of $\mathbf{R}'_O$. If A^V is any V-formula of $\mathbf{R}'_O$, then the O-transform of A^V is the formula A^O got by replacing every part of A^V of the form $\neg B \to V$ by OB. Evidently if A^V is a V-formula of $\mathbf{R}'_O$, then A^O will be a formula of $\mathbf{R}_O$ (as well as $\mathbf{R}'_O$).

Observation 2.4 If A^V is a V-formula of $\mathbf{R}'_O$ and A^O is its O-transform, then $\vdash A^V$ in $\mathbf{R}'_O$ if and only if $\vdash A^O$ in $\mathbf{R}_O$.

Proof[1] We first observe that in $\mathbf{R}'_O$ we have $(\neg B \to V) \leftrightarrow OB$ and a derivable rule of substitution, so $\vdash A^V$ in $\mathbf{R}'_O$ if and only if $\vdash A^O$ in $\mathbf{R}'_O$. This is half the battle; what remains to be proven is that $\mathbf{R}'_O$ is a conservative extension of $\mathbf{R}_O$, that is, that each V-free formula of $\mathbf{R}'_O$ has a V-free proof. Such a proof will also be a proof in $\mathbf{R}_O$, from which it will follow that if $\vdash A^O$ in $\mathbf{R}'_O$ then $\vdash A^O$ in $\mathbf{R}_O$.

The leading idea is that, although V cannot be replaced by the same V-free formula in every proof, it is still possible to find for each proof of a V-free formula, a particular V-free formula that can replace V throughout that proof.

Let $A_1, \ldots, A_n$ ($A_n = A$) be a proof of A in $\mathbf{R}'_O$, and let $p_1, \ldots, p_m$ be a list of all propositional variables occurring in the proof $A_1, \ldots, A_n$. Then, for this proof of A, we define $\mathbf{t}$ as $\&_{i=1}^{m}(p_i \to p_i)$, $\mathbf{f}$ as $\neg\mathbf{t}$, and V' as $O\mathbf{f}$.

Let A'_i be the result of replacing V throughout A_i by V'. We show inductively that each of $A'_1, \ldots, A'_n$ ($= A$) has a V-free proof in $\mathbf{R}'_O$, which is to say a proof in $\mathbf{R}_O$, as required.

(i) If A_i is one of the axioms R1, …, R11, D1, D2 of $\mathbf{R}'_O$, then $\vdash A'_i$ in $\mathbf{R}_O$ by the same axiom.

(ii) If A_i is an axiom D3 of $\mathbf{R}'_O$, then A'_i has the form $OB \to (\neg B \to V')$. We need to show that A'_i is provable in $\mathbf{R}_O$. Let $q_1, \ldots, q_k$ be all the variables occurring in B. Then an easy induction on the length of B shows that $\vdash \&_{j=1}^{k}(q_j \to q_j) \to (B \to B)$. Evidently, $\vdash \&_{i=1}^{m}(p_i \to p_i) \to \&_{j=1}^{k}(q_j \to q_j)$ since the q_j are all among the p_i, so $\vdash \mathbf{t} \to (B \to B)$ by def. $\mathbf{t}$, hence $\vdash B \to (\mathbf{t} \to B)$ by permutation, hence $\vdash B \to (\neg B \to \mathbf{f})$ by T2 and def. $\mathbf{f}$, hence $\vdash \neg B \to (B \to \mathbf{f})$ by permutation, hence $\vdash \neg B \to (OB \to O\mathbf{f})$ by D1, hence $\vdash OB \to (\neg B \to O\mathbf{f})$ by permutation, hence $\vdash OB \to (\neg B \to V')$ by def. V', as desired.

(iii) If A_i is an axiom D4 of $\mathbf{R}'_O$, then A'_i has the form $(\neg B \to V') \to OB$. We need to show that A'_i is provable in $\mathbf{R}_O$. $\vdash (O\mathbf{f} \to \mathbf{f}) \to (\mathbf{t} \to \neg O\mathbf{f})$ by contraposition and def. $\mathbf{f}$, hence $\vdash \mathbf{t} \to ((O\mathbf{f} \to \mathbf{f}) \to \neg O\mathbf{f})$ by permutation, hence $\vdash (O\mathbf{f} \to \mathbf{f}) \to \neg O\mathbf{f}$ by $\vdash \mathbf{t}$ (from def. $\mathbf{t}$, self-implication, and adjunction), hence $\vdash (\neg O\mathbf{f} \to B) \to ((O\mathbf{f} \to \mathbf{f}) \to B)$ by T1, hence $\vdash (\neg O\mathbf{f} \to B) \to (O(O\mathbf{f} \to \mathbf{f}) \to OB)$ by D1, hence $\vdash O(O\mathbf{f} \to \mathbf{f}) \to ((\neg O\mathbf{f} \to B) \to OB)$ by permutation, hence $\vdash (\neg O\mathbf{f} \to B) \to OB$ by D2, hence $\vdash (\neg B \to O\mathbf{f}) \to OB$ by T2 and double negation, hence $\vdash (\neg B \to V') \to OB$ by def. V', as desired.

(iv) If A_i is a conclusion from premises A_j and A_k by detachment or adjunction, then $\vdash A'_j$ and $\vdash A'_k$ in $\mathbf{R}_O$ by the inductive hypothesis, and $\vdash A'_i$ in $\mathbf{R}_O$ by the same rule.

This completes the proof, which shows essentially that the addition of V and axioms D3 and D4 is otiose, since $\mathbf{R}_O$ already contains an equivalent deontic theory. □

Definition 2.5 Anderson's relevant deontic logic $\mathbf{R}_V$ is $\mathbf{R}$ plus a primitive propositional constant V and the following definition of O: $OA = \neg A \to V$.

Observation 2.6 $\vdash A$ in $\mathbf{R}_V$ if and only if $\vdash A'$ in $\mathbf{R}'_O$, where A' is the result of replacing V throughout A by V'.

Proof By induction on the derivation of A or A'. Axioms D3 and D4 and a derivable rule of substitutability of provable equivalents may be used instead of the definition of O, and vice versa. □

Observation 2.7 If A^V is a V-formula of $\mathbf{R}_V$ and A^O is its O-transform, then $\vdash A^V$ in $\mathbf{R}_V$ if and only if $\vdash A^O$ in $\mathbf{R}_O$.

Proof From Observations 2.4 and 2.6. □

Definition 2.8 The *deontic fragment* of $\mathbf{R}_V$ is the set $\{A^O : \vdash A^V \text{ in } \mathbf{R}_V\}$.

Observation 2.9 $\mathbf{R}_O$ is an axiomatization of the deontic fragment of $\mathbf{R}_V$.

Proof Immediate from Observation 2.7. □

Observation 2.9 was also made in Lokhorst [6], which, however, was based on the needlessly complicated semantical proof in [5].

Definition 2.10 $\mathbf{R}_O$**.a** is $\mathbf{R}_O$ plus the axiom $OA \to \neg O \neg A$ (Anderson called this the "axiom of avoidance"). $\mathbf{R}_V$**.a** is $\mathbf{R}_V$ plus the axiom $\neg(\neg V \to V)$.

Observation 2.11 $\mathbf{R}_O$**.a** is an axiomatization of the deontic fragment of $\mathbf{R}_V$**.a**.

Proof By obvious extensions of the proof of Observation 2.9. It is easily proven that $\vdash \neg(\neg V' \to V')$ in $\mathbf{R}'_O$**.a** and that $\vdash OA \to \neg O \neg A$ in $\mathbf{R}_V$**.a**. □

All these results also hold for the corresponding *linear* rather than relevant deontic systems because contraction and distribution are not needed.

Observation 2.12 Axiom D2 of $\mathbf{R}_O$ may be replaced with either of the following two axioms:

D2′ $A \to O\neg O \neg A$;
D2″ $(A \to OB) \to O(A \to B)$.

Proof First, D2′ and D2″ are theorems of $\mathbf{R}_O$: see [6], §3. Second, if axiom D2 is replaced with D2′, then D2 becomes a theorem:

1. $\vdash \neg A \to (A \to \neg(A \to A))$ by $\mathbf{R}$, hence
2. $\vdash \neg A \to (OA \to O\neg(A \to A))$ by D1, hence
3. $\vdash \neg A \to (\neg O \neg(A \to A) \to \neg OA)$ by $\mathbf{R}$, hence
4. $\vdash \neg O \neg(A \to A) \to (\neg A \to \neg OA)$ by permutation, hence
5. $\vdash O\neg O \neg(A \to A) \to O(\neg A \to \neg OA)$ by D1, hence
6. $\vdash O(\neg A \to \neg OA)$ by self-implication and D2′, hence
7. $\vdash O(OA \to A)$ [=D2] by $\mathbf{R}$ and D1.

Third, D2″ yields D2 by self-implication. □

Definition 2.13 It is forbidden that A: $FA = O\neg A$. It is permitted that A: $PA = \neg O \neg A$. It is elective (optional) that A: $EA = \neg OA$. A does not exclude B: $A \circ B = \neg(A \to \neg B)$.

The relations between the four resulting deontic concepts are illustrated in the square of opposition shown in Figure 1. The axiom of avoidance says that $\vdash OA \to PA$

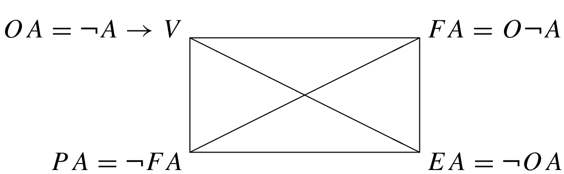

Figure 1 Four deontic concepts ([3], Fig. 6).

and $\vdash FA \to EA$.

Observation 2.14 $\mathbf{R}_O$ can also be axiomatized in either of the following three ways:

1. $\mathbf{R}$ plus $(A \to B) \to (FB \to FA)$ and $A \to FFA$;
2. $\mathbf{R}$ plus $(A \to B) \to (PA \to PB)$ and $P(A \circ B) \to (A \circ PB)$;
3. $\mathbf{R}$ plus $(A \to B) \to (EB \to EA)$ and $EEA \to A$.

Proof From Observation 2.12 and Definition 2.13. □

$\mathbf{R}_O$ is *inadequate* as a system of deontic logic. For example, D2 is intuitively acceptable while D2′ ("everything that is the case ought to be permitted"; "if there is slavery, it is forbidden to forbid slavery") is unacceptable: yet D2 yields D2′ by D1. This means that D1 has to be rejected. It is better to replace it with the *rule of inference* $A \to B/OA \to OB$. See [5], §3ff., and [6], §7ff., for relevant deontic systems along these lines.

3 Anderson's Relevant Eubouliatic Logic

Definition 3.1 Eubouliatic system $\mathbf{R}_{R_w}$ has the same language as $\mathbf{R}$ except that there is an additional primitive unary connective R_w, read as "it is without risk that" or "it is safe that." $\mathbf{R}_{R_w}$ has the following axioms in addition to those of $\mathbf{R}$:

E1 $(A \to B) \to (R_w B \to R_w A)$,
E2 $A \to R_w R_w A$.

Definition 3.2 Eubouliatic system $\mathbf{R}'_{R_w}$ is $\mathbf{R}_{R_w}$ plus a constant G ("the good thing") and the following axioms:

E3 $R_w A \to (A \to G)$,
E4 $(A \to G) \to R_w A$.

We refer to those formulas of $\mathbf{R}'_{R_w}$ in which G occurs, if at all, only in contexts of the form $A \to G$ (so that G is always eliminable in terms of R_w) as G-formulas of $\mathbf{R}'_{R_w}$. If A^G is any G-formula of $\mathbf{R}'_{R_w}$, then the R_w-transform of A^G is the formula A^{R_w} got by replacing every part of A^G of the form $B \to G$ by $R_w B$.

Observation 3.3 If A^G is a G-formula of $\mathbf{R}'_{R_w}$ and A^{R_w} is its R_w-transform, then $\vdash A^G$ in $\mathbf{R}'_{R_w}$ if and only if $\vdash A^{R_w}$ in $\mathbf{R}_{R_w}$.

Proof The proof proceeds along the same lines as the proof of Observation 2.4, but it is simpler because negation plays no role. As above, the "if" part of the proof is easy. For the "only if" part, we define $\mathbf{t}$ as above and G' as $R_w\mathbf{t}$. The inductive proof is modified as follows. (i) As above. (ii) $\vdash B \to (\mathbf{t} \to B)$ as above, hence $\vdash B \to (R_w B \to R_w\mathbf{t})$ by E1, hence $\vdash R_w B \to (B \to G')$ by permutation and def. G', as desired. (iii) $\vdash (B \to R_w\mathbf{t}) \to (R_w R_w\mathbf{t} \to R_w B)$ by E1, hence $\vdash R_w R_w \mathbf{t} \to ((B \to R_w\mathbf{t}) \to R_w B)$ by permutation, hence $\vdash (B \to R_w\mathbf{t}) \to R_w B$ by $\vdash \mathbf{t}$ and E2, hence $\vdash (B \to G') \to R_w B$ by def. G', as desired. (iv) As above. □

Definition 3.4 Anderson's relevant eubouliatic logic $\mathbf{R}_G$ is $\mathbf{R}$ enriched with G and the definition $R_w A = A \to G$ ("A guarantees the good thing"). The *eubouliatic fragment* of $\mathbf{R}_G$ is the set $\{A^{R_w} : \ \vdash A^G \text{ in } \mathbf{R}_G\}$.

Observation 3.5 $\mathbf{R}_{R_w}$ is an axiomatization of the eubouliatic fragment of $\mathbf{R}_G$.

Proof From Observation 3.3, along the same lines as the proof of Observation 2.9. □

This result also holds for the corresponding *positive* and *linear* systems because negation, contraction, and distribution play no role.

Definition 3.6 $\mathbf{R}_{R_w}$**.a** is $\mathbf{R}_{R_w}$ plus the "axiom of avoidance" $R_wA \to \neg R_w\neg A$. $\mathbf{R}_G$**.a** is $\mathbf{R}_G$ plus the axiom $\neg(\neg G \to G)$.

Observation 3.7 $\mathbf{R}_{R_w}$**.a** is an axiomatization of the eubouliatic fragment of $\mathbf{R}_G$**.a**.

Proof By obvious extensions of the proof of Observation 3.5. □

Definition 3.8 $HA = R_w\neg A$, $CA = \neg R_w\neg A$, and $RA = \neg R_wA$.

Anderson read HA as "it is heedless that A," CA as "it is cautious that A," and RA as "it is risky that A," but he stressed that he was "far from satisfied with the[se] terminological choices" ([3], p. 279).

The relations between the four resulting eubouliatic concepts are illustrated in the square of opposition shown in Figure 2. The axiom of avoidance says that

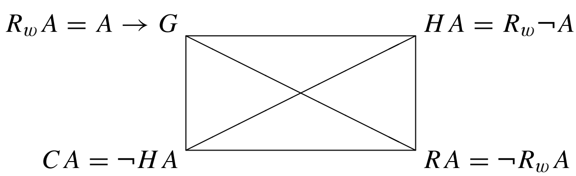

Figure 2 Four eubouliatic concepts ([3], Fig. 8).

$\vdash R_wA \to CA$ and $\vdash HA \to RA$.

Observation 3.9 $\mathbf{R}_{R_w}$ can also be axiomatized in either of the following three ways:

1. $\mathbf{R}$ plus $(A \to B) \to (HA \to HB)$ and $H(HA \to A)$;
2. $\mathbf{R}$ plus $(A \to B) \to (CB \to CA)$ and $CCA \to A$;
3. $\mathbf{R}$ plus $(A \to B) \to (RA \to RB)$ and $R(A \circ B) \to (A \circ RB)$.

Proof Similar to the proof of Observation 2.14 because $\mathbf{R}_{R_w}$ is just a relettered version of $\mathbf{R}_O$ ($F \mapsto R_w$, $O \mapsto H$, $E \mapsto C$, $P \mapsto R$). □

4 The Logic of Safety

Anderson's notion of safety is different from the notion of safety in ordinary language. For example, $\mathbf{R}_{R_w}$ has the theorem $R_wA \to R_w(A \mathbin{\&} B)$, but we do not normally say that if it is safe that John drinks a glass of water, then it is also safe that John drinks a glass of water and detonates a bomb.

Anderson explained his conception of safety as follows.

> I suppose that we should agree in calling events or states-of-affairs which ensure avoidance of the bad state-of-affairs "prudent," or perhaps "safe." Actually what is meant is that such a state-of-affairs (or proposition) p is *without risk*, in the sense that the proposition *guarantees* that no trouble will ensue.

> Of course no one supposes that this is a logical guarantee, or even an empirical one; it is as easy to make logical mistakes in practice as it is to be run over by a bus. But the formal logic of the present logical situation is still, I claim, clear to all of us. We all know perfectly well that the rules of chess entail that the opening player may make only one move before his opponent has a turn. We also know that the rules permit exactly one of twenty possible opening moves, any one of which leads to a position on the board describable by a sentence (e.g. "A white knight is at KR3 and all other pieces are in their initial positions") expressing a proposition (in this case the proposition that a white knight is at KR3 and all other pieces are in their initial positions). Moreover, any of these twenty propositions may be true without risk of violating the rules of chess, so that for any such proposition p we have
>
> $$p \to \neg V.$$
>
> For reasons which will emerge shortly, I shall abbreviate this as
>
> $$R_w p,$$
>
> with some such unidiomatic, but still reasonably unambiguous, interpretation as "it is without risk that p," or (again ignoring the use-mention distinction) "p is riskless," or "p is safe." The reader however *must* bear in mind that though the word "risk" has prudential, or utilitarian, or strategic connotations in English, the only risk in question here is a *risk of violating the rules* defining the practice in question, and *not* a risk of e.g. making a strategically poor move. ([3], pp. 275–6)

In other words, Anderson regarded the proposition that p (where p means that there is a white knight at KR3 and that all other pieces are in their initial positions) as safe in the sense that p guarantees that the rules are not violated.

However, p is *not* safe in this sense. Suppose that q means that a couple of spare kings are added to the board. $p \,\&\, q$ guarantees that p, so if p guarantees that the rules are not violated, then $p \,\&\, q$ does so too. But this is absurd because it is clear that $p \,\&\, q$ violates the rules. So p does *not* guarantee that the rules are not violated; p is *not* safe in Anderson's sense of "it is safe that."

The proposition that p is, however, safe in another sense of the word: it is safe in the sense that the rules do not exclude it. As Anderson put it, "the rules permit exactly one of twenty possible opening moves." The proposition that John drinks a glass of water is safe in the same sense: the good thing (surviving the day, say) does not exclude it. The proposition that John drinks a glass of water and detonates a bomb, on the other hand, is not safe in this sense: the good thing rules it out and it guarantees disaster.

We therefore propose the following alternative analysis of safety and related notions. We assume that $G = \neg V$.

1. It is safe that A: $SA = \neg(A \to V)$ ("A does not guarantee the bad thing"). Note that $\vdash SA \leftrightarrow (A \circ G)$ and that $\vdash SA \leftrightarrow (G \circ A)$. (Safety is compatibility with the good thing rather than a guarantee for the good thing.)
2. It is unsafe (disastrous) that A: $UA = \neg SA = A \to V = \neg(A \circ G)$.
3. It is mandatory (essential) that A: $MA = \neg A \to V = G \to A$.
4. It is inessential that A: $IA = \neg MA = \neg(\neg A \to V) = \neg A \circ G$.

These notions are illustrated in Figure 3. An axiom of avoidance would say that

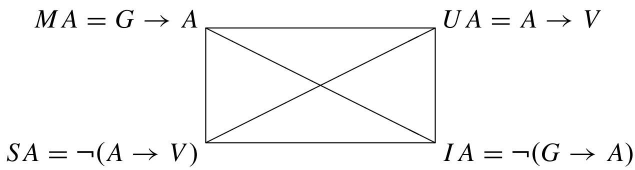

Figure 3 Safety and related concepts.

$\vdash MA \to SA$ and $\vdash UA \to IA$.

The "eubouliatic fragment" (in the obvious sense) of the new system can be axiomatized in either of the following four ways:

1. **R** plus $(A \to B) \to (MA \to MB)$ and $M(MA \to A)$;
2. **R** plus $(A \to B) \to (UB \to UA)$ and $A \to UUA$;
3. **R** plus $(A \to B) \to (SA \to SB)$ and $S(A \circ B) \to (A \circ SB)$;
4. **R** plus $(A \to B) \to (IB \to IA)$ and $IIA \to A$.

The proof is similar to the proof of Observations 2.9 and 2.14 because the new system is just a relettered version of $\mathbf{R}_O$ $(O \mapsto M, F \mapsto U, P \mapsto S, E \mapsto I)$.

The new system does not have the theorem $SA \to S(A \,\&\, B)$, so it seems to give a more adequate account of safety than Anderson's system did, at least in the particular examples that we have discussed. But we cannot conclude from this that it is generally more adequate. The basic problem is that safety is a complicated and unclear concept with many connotations. The *Oxford English Dictionary* describes no less than eleven different senses of "safety" [7]. Causal, epistemic, modal, probabilistic, and temporal notions all seem to play some role. It would be unrealistic to expect that we can do full justice to such a complicated concept in the austere language of propositional logic enriched with nothing but a single propositional constant.

Note

1. This proof is modeled after Anderson et al. [4], ch. 8, §45.1, which closely follows Anderson and N. D. Belnap [1], §2.

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